S.R. Study Material

S R SAMPLE PAPER 3

Class 12 - Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If $A_\alpha = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ then $(A_\alpha)^2$ = ? **[1]**

a) $\begin{bmatrix} \cos^2\alpha & \sin^2\alpha \\ -\sin^2\alpha & \cos^2\alpha \end{bmatrix}$

b) $\begin{bmatrix} 2\cos\alpha & 2\sin\alpha \\ -\sin\alpha & 2\cos\alpha \end{bmatrix}$

c) None of these

d) $\begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}$

2. The value of the determinant of a skew symmetric matrix of even order is **[1]**

a) A non zero perfect square

b) None of these

c) 0

d) Negative

3. If A is a square matrix such that (A - 2I)(A + I) = 0, then A^{-1} is **[1]**

a) 2(A - I)

b) 2(A + I)

c) $\frac{A+I}{2}$

d) $\frac{A-I}{2}$

4. Let $f(x) = \begin{cases} \frac{1}{|x|} & \text{for } |x| \geq 1 \\ ax^2 + b & \text{for } |x| < 1 \end{cases}$ If f(x) is continuous and differentiable at any point, then **[1]**

a) a = 1, b = –1

b) $a = \frac{1}{2}, b = -\frac{3}{2}$

c) $a = \frac{1}{2}, b = \frac{3}{2}$

d) none of these

5. Find the shortest distance between the lines whose vector equations are $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda\left(\hat{i} - 3\hat{j} + 2\hat{k}\right)$ and $\vec{r} = 4\hat{i} + 5\hat{j} + 6\hat{k} + \mu\left(2\hat{i} + 3\hat{j} + \hat{k}\right)$ **[1]**

a) $\frac{3}{\sqrt{17}}$

b) $\frac{3}{\sqrt{23}}$

c) $\frac{3}{\sqrt{27}}$ d) $\frac{3}{\sqrt{19}}$

6. For the differential equation $xy\frac{dy}{dx} = (x+2)(y+2)$ find the solution curve passing through the point (1, –1). **[1]**

a) $y + x + 2 = log\left(x^2(y+2)^2\right)$
b) $y - x - 2 = log\left(x^2(y-2)^2\right)$
c) $y - x + 2 = log\left(x^2(y+2)^2\right)$
d) $y - x - 2 = log\left(x^2(y+2)^2\right)$

7. The value of objective function is maximum under linear constraints **[1]**

a) at (0, 0)
b) at any vertex of feasible region
c) the vertex which is maximum distance from (0, 0)
d) at the centre of feasible region

8. Let a vector $\vec{r}$ make angles 60°, 30° with X and Y-axes, respectively. What are the direction cosines of $\vec{r}$? **[1]**

a) $< \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 >$
b) $< \frac{1}{2}, \frac{\sqrt{3}}{2}, 0 >$
c) $< -\frac{1}{2}, \frac{\sqrt{3}}{2}, 0 >$
d) $< \frac{1}{2}, -\frac{\sqrt{3}}{2}, 0 >$

9. If $\int_0^a \frac{1}{1+4x^2}dx = \frac{\pi}{8}$, then a equals **[1]**

a) 1
b) $\frac{\pi}{4}$
c) $\frac{1}{2}$
d) $\frac{\pi}{2}$

10. If (2A - B) $= \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}$ and (2B + A) $= \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix}$ then A = ? **[1]**

a) None of these
b) $\begin{bmatrix} -3 & 2 & 1 \\ 2 & 1 & -1 \end{bmatrix}$
c) $\begin{bmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{bmatrix}$
d) $\begin{bmatrix} 3 & 2 & -1 \\ 2 & -1 & 1 \end{bmatrix}$

11. Which of the following is not a convex set? **[1]**

a) [(x, y) : 2x + 5Y < 7]
b) $\left\{(x, y) : x^2 + y^2 \leq 4\right\}$
c) {X: |X| = 5}
d) {(x, y) : $3x^2 + 2y^2 \leq 6$)

12. ABCD is a parallelogram and P is the point of intersection of the diagonals. If O is the origin, then OA + OB + OC + OD is equal to **[1]**

a) OP
b) 4OP
c) 2OP
d) Null vector

13. If I_3 is the identity matrix of order 3, then I_3^{-1} is **[1]**

a) 0
b) $3I_3$
c) I_3
d) None of these

14. If E and F are independent, then _____ **[1]**

a) P (E ∩ F) = P (E) P (E|F)
b) P (E ∩ F) = P (E) P (F)
c) P (E ∩ F) = P (E) P (F|E)
d) P (E ∩ F) =P (E ∪ F)

15. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 - \left(\frac{dy}{dx}\right) = y^3$, is **[1]**

a) 4
b) $\frac{1}{2}$

c) 2 d) 3

16. Two adjacent sides of a parallelogram are represented by the vectors $\vec{a}=(3\hat{i}+\hat{j}+4\hat{k})$ and $\vec{b}=(\hat{i}-\hat{j}+\hat{k})$ the area of the parallelogram is **[1]**

a) 6 sq units
b) $\sqrt{42}$
c) none of these
d) $\sqrt{35}$

17. Let g(x) = $\begin{cases} e^{2x}, & x<0 \\ e^{-2x}, & x\geq 0 \end{cases}$ then g(x) does not satisfy the condition **[1]**

a) none of these
b) continuous $\forall$ x $\in$ R
c) continuous $\forall$ x $\in$ R and non differentiable at x = ± 1
d) not differentiable at x = 0

18. The vector equation of the x-axis is given by **[1]**

a) $\vec{r}=\hat{j}+\hat{k}$
b) none of these
c) $\vec{r}=\hat{i}$
d) $\vec{r}=\lambda\hat{i}$

19. **Assertion (A):** If x is real, then the minimum value of x^2 - 8x + 17 is 1. **[1]**
Reason (R): If f"(x) > 0 at a critical point, then the value of the function at the critical point will be the minimum value of the function.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** The relation R in the set A = (1, 2, 3, 4) defined as R = {(x, y): y is divisible by x) is an equivalence relation. **[1]**
Reason (R): A relation R on the set A is equivalence if it is reflexive, symmetric and transitive.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. Find the principal value of $\text{cosec}^{-1}(-2)$. **[2]**

OR

Evaluate: $\sin^{-1}(\sin(-600°))$

22. Find the interval in which the function f(x) = $x^3 - 6x^2 - 36x + 2$ is increasing or decreasing. **[2]**

23. An edge of a variable cube is increasing at the rate of 10 cm/sec. How fast the volume of the cube is increasing when the edge is 5 cm long? **[2]**

OR

Show that function $x^2 - x + 1$ is neither increasing nor decreasing on (0,1)

24. Evaluate: $\int \sin\sqrt{x}dx$ **[2]**

25. A man 160 cm tall, walks away from a source of light situated at the top of a pole 6 m high, at the rate of 1.1 m/sec. How fast is the length of his shadow increasing when he is 1 m away from the pole? **[2]**

Section C

26. Evaluate: $\int_0^{\pi/2} \frac{\cos x}{1+\cos x+\sin x} dx$ **[3]**

27. Three persons A, B and C apply for a job of Manager in a private company. Chances of their selection (A, B and C are in the ratio 1:2:4. The probabilities that A, B and C can introduce changes to improve profits of the company are 0.8, 0.5 and 0.3, respectively. If the change does not take place, find the probability that it is due to the appointment of C. **[3]**

28. Evaluate: $\int_0^a \sqrt{a^2 - x^2} dx$ **[3]**

OR

Evaluate $\int \frac{\sin(x-a)}{\sin(x+a)} dx.$

29. Find the particular solution of the differential equation $x\frac{dy}{dx} + y + \frac{1}{1+x^2}$ = 0, given that y(1) = 0. **[3]**

OR

Find the particular solution of the differential equation $\cos x \frac{dy}{dx} + y = \sin x$, given that y = 2 when x = 0.

30. Solve the following LPP by graphical method: **[3]**

Minimize Z = 20x + 10y

Subject to

x + 2y $\leq$ 40

3x + y $\geq$ 30

4x + 3y $\geq$ 60

and x, y $\geq$ 0

OR

Maximize Z = 100x + 170y subject to

3x+2y$\leq$ 3600

x+4y$\leq$1800

$x \geq 0, y \geq 0$

31. If x = a$\left(\cos t + \log \tan \frac{t}{2}\right)$, y = a sin t, then evaluate $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{3}$. **[3]**

Section D

32. If the area bounded by the parabola $y^2 = 16ax$ and the line $y = 4mx$ is $\frac{a^2}{12}$ sq. units, then using integration, find the value of m. **[5]**

33. Let A and B be two sets. Show that f: A $\times$B $\rightarrow$ B $\times$A such that f(a, b) = (b, a) is a bijective function. **[5]**

OR

Let A = R - {3} and B = R - {1}. Consider the function f : A $\rightarrow$ B defined by $f(x) = \left(\frac{x-2}{x-3}\right)$ Is f one-one and onto? Justify your answer.

34. Two schools P and Q want to award their selected students on the values of Tolerance, Kindness, and Leadership. The school P wants to award Rs x each, Rs y each and Rs z each for the three respective values to 3, 2 and 1 students respectively with total award money of Rs2200. **[5]**

School Q wants to spend Rs 3100 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as school P). If the total amount of award for one prize on each value is Rs1200, using matrices, find the award money for each value.

35. Find the shortest distance between the given lines. $\vec{r} = (6\hat{i} + 3\hat{k})+\lambda(2\hat{i} - \hat{j} + 4\hat{k})$, $\vec{r} = (-9\hat{i} + \hat{j} - 10\hat{k})$ $+\mu(4\hat{i} + \hat{j} + 6\hat{k})$ **[5]**

OR

Find the image of the point (5, 9, 3) in the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Akash and Prakash appeared for first round of an interview for two vacancies. The probability of Nisha's selection is $\frac{1}{3}$ and that of Ayushi's selection is $\frac{1}{2}$.

(i) Find the probability that both of them are selected.

(ii) The probability that none of them is selected.

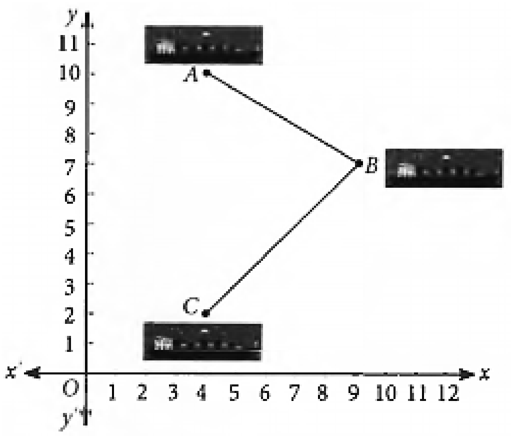
(iii) Find the probability that only one of them is selected.

OR

Find the probability that atleast one of them is selected.

37. **Read the text carefully and answer the questions:** **[4]**

A barge is pulled into harbour by two tug boats as shown in the figure.



(i) Find position vector of A.

(ii) Find position vector of B.

(iii) Find the vector $\overrightarrow{AC}$ in terms of $\hat{i}, \hat{j}$.

OR

If $\vec{A} = 4\hat{i} + 3\hat{j}$ and $\vec{B} = 3\hat{i} + 4\hat{j}$, then find $|\vec{A}| + |\vec{B}|$

38. **Read the text carefully and answer the questions:** **[4]**

In an elliptical sport field, the authority wants to design a rectangular soccer field with the maximum possible area. The sport field is given by the graph of $\frac{x^2}{a^2} + \frac{y^2}{b^2}$ = I

(i) If the length and the breadth of the rectangular field be 2x and 2y respectively, then find the area function in terms of x.

(ii) Find the critical point of the function.